Team Round

- **DO NOT open this test until your proctor tells you to begin.**
- This portion of the contest consists of 12 problems that are to be completed in 30 minutes, along with a small minigame that will generate a multiplier for a team's team round score.
- There is no partial credit or penalty for incorrect answers.
- The minigame, or problem 13, will ask your team to submit a rational number, and will generate a multiplier between 0.9 and 1.1 for your team round score. Be sure to try out the minigame, as submitting nothing will decrease your team round score.
- Each of the 12 team round problems have a predetermined point value, indicated next to the problem numbers; your team's team round score will be the sum of the point values assigned to each question that is correctly answered, multiplied by the multiplier generated by the minigame. Excluding the multiplier, a perfect score (achieved through answering all 12 problems correctly) is **400 points**.
- Your team score will be a combination of your score on the team round and the scores of each individual member.
- No aids other than the following are permitted: scratch paper, graph paper, ruler, compass, protractor, writing utensils, and erasers. **No calculators or other electronic devices (including smartwatches) are permitted.**
- Please make sure to record your team name, team members' names, and all answers on your answer form. Only the responses on the answer forms will be graded.
- All answers are of one of the following forms. Please follow these instructions for converting your answer into an integer, so all answers submitted to the Google form are integers.
  - a for an integer a
    - Submit a
  - $\frac{a}{b}$ for relatively prime positive integers a, b with $b > 1$
    - Submit $a + b$
  - $a\pi + b$ for integers a, b with $a \neq 0$
    - Submit $a + b$
  - $\sqrt{a}$ for a positive nonsquare positive integer a
    - Submit a
  - $\frac{a\sqrt{b}}{c}$ for a squarefree positive integer b, positive integer $c > 1$, and $\gcd(a, c) = 1$
    - Submit $a + b + c$

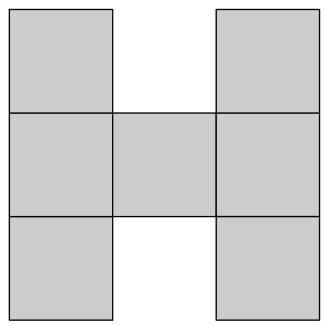
1. **[22]** Let c be a real number. Suppose that the line with equation $y = cx$ is perpendicular to the line with equation $x + \frac{y}{8} = 2cx + 1$. What is the value of c?

2. **[24]** Two unit circles are given such that their common internal tangent has half the length of their common external tangent. Find the distance between their centers.

3. **[26]** Ethan has 5 different books: one yellow, two green, and two red. If the yellow book is between two books of the same color (but not necessarily adjacent to these two books), how many ways can he arrange the books on his shelf?

4. **[28]** The cells of a 12×12 multiplication table are filled in such that the cell in the ith row and jth column is labeled with the product $i \cdot j$. How many of these products end with a 0?

5. **[30]** Holden makes the letter H, a connected figure with seven blocks, as depicted. He then removes blocks one at a time without ever disconnecting the figure until all blocks are removed. Two blocks that only touch at a corner are not considered connected, so for example, Holden cannot remove the middle block or the blocks to its left or right on the first turn. How many different ways can he do this?



6. **[32]** Let $ABCD$ be a rectangle with diagonals $AC = BD = 12$. Similar isosceles triangles EAB, FBC, GCD, HDA with bases AB, BC, CD, DA respectively are drawn outside the rectangle and have total area 24. If the areas of $ABCD$ and $EFGH$ are equal, find this common area.

7. **[34]** A sequence of six prime numbers is given such that

   - the first term has two digits, and
   - each term is one more than twice its previous term.

   What is the last term?

8. **[36]** A polygon with all angles equal to $90°$ or $270°$ has the property that its sides have distinct prime lengths. Find the minimum possible perimeter.

9. **[38]** Let x, y, z be nonzero complex numbers such that $x + y + z = 5$, $x^2y + y^2z + z^2x = 20$, and $xy^2 + yz^2 + zx^2 = 19$. Compute
$$\frac{x^3 + y^3 + z^3 - 8}{xyz}.$$

10. **[40]** Let P be a point on the incircle ω of unit equilateral triangle ABC. Let X be the second intersection of ω and $\overline{AP}$ and Y be the second intersection of ω and $\overline{BP}$. Find the maximum possible area of triangle PXY.

11. **[42]** Let α be a real number satisfying $\alpha^3 = \alpha + 1$. Find $m^2 + n^2$ if $m < n$ are integers satisfying $\alpha^m + \alpha^n = 2$.

12. **[44]** Find the smallest positive integer with the property that the product of its prime divisors is equal to the sum of its nonprime proper divisors (a proper divisor is a positive integer divisor of a number, excluding itself).

13. Let X be the fraction of teams that will submit an answer less than $\frac{1}{2}$ to this problem. Estimate X. Your multiplier will be $0.9 + 0.2 \times \left(\min\left\{\frac{X}{Y}, \frac{Y}{X}\right\}\right)^2$, where Y is the answer you submit. If you submit nothing, an irrational number, or a number outside the range $(0, 1]$, your multiplier will default to 0.9.