Individual Round

- **DO NOT open this test until your proctor tells you to begin.**
- This portion of the contest consists of 20 problems that are to be completed in 60 minutes.
- There is no partial credit or penalty for incorrect answers.
- Each question will be weighted after the contest window according to the percentage of correct answers, and your individual score will be the sum of the point values assigned to each question that is correctly answered. A perfect score (achieved through answering all 20 problems correctly) is **200 points**.
- No aids other than the following are permitted: scratch paper, graph paper, ruler, compass, protractor, writing utensils, and erasers. **No calculators or other electronic devices (including smartwatches) are permitted.**
- Please make sure to record your name, school, and all answers on your answer form. Only the responses on the answer forms will be graded.
- All answers are of one of the following forms. Please follow these instructions for converting your answer into an integer, so all answers submitted to the Google form are integers.
  - a for an integer a
    - Submit a
  - $\frac{a}{b}$ for relatively prime positive integers a, b with $b > 1$
    - Submit $a + b$
  - $a\pi + b$ for integers a, b with $a \neq 0$
    - Submit $a + b$
  - $\sqrt{a}$ for a positive nonsquare positive integer a
    - Submit a
  - $\frac{a\sqrt{b}}{c}$ for a squarefree positive integer b, positive integer $c > 1$, and $\gcd(a, c) = 1$
    - Submit $a + b + c$

1. Sara writes the equation
$$35 + 6z = 71$$
on a whiteboard for Zoe to solve. Since Zoe isn't wearing her glasses, the 5 looks like an s and the z looks like a 2. What is the positive difference between Sara's answer for z and Zoe's answer for s?

2. Toto has 10 boxes each containing 5 beans and 5 boxes each containing 10 beans. Each box contains one pinto bean. If Toto selects a box at random, and then selects a bean from that box at random, what is the probability that he selects a pinto bean?

3. In triangle GDL, I and E lie on sides GD, GL respectively such that $GI = 5$, $ID = 11$, $EG = 8$, $EL = 2$, and $IE = 7$. What is DL?

4. Compute
$$\frac{1}{\frac{1}{41+9} + \frac{1}{41-9}} + \frac{1}{\frac{1}{41+40} + \frac{1}{41-40}}.$$

5. Mamamoo is a K-pop girl group consisting of the members Solar, Moonbyul, Wheein, and Hwasa. In their song "Wind Flower", the members sing one at a time for a total of 4 minutes of singing. Moonbyul sings for a third of amount of time that Wheein and Hwasa collectively sing. Given that Moonbyul sings for the least amount of time, what is the greatest possible difference between the number of seconds that Wheein and Hwasa sing?

6. How many ways are there to arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8 in a circle such that no two adjacent numbers share a common factor greater than 1? Two arrangements are considered to be the same if they are rotations of each other.

7. A class of students took a test, each receiving an integer grade from 0 to 10 inclusive. If the average score of the top 25 students was $9\frac{1}{5}$, and the average score of all the students was $6\frac{2}{7}$, what is the smallest possible number of students?

8. You and five of your friends line up in a random order to get tickets, one at a time. There are sixteen tickets available, labelled 1 through 16. Your friends do not care which ticket they get, and will take one at random; but your favorite number is 13, so if ticket #13 is available, you will take it. What is the probability that you get ticket #13?

9. An acute triangle and a circle centered at the triangle's incenter both have an area of 4π. Given that $\frac{3}{4}$ of the circle's circumference lies outside the triangle, find the total area of the regions that are inside the triangle and outside the circle.

10. Let $ABCD$ be a rectangle, and let G be the centroid of ABC. Given that $DA = DG$, find $\frac{AD}{CD}$.

11. Call a positive integer *redundant* if all of its digits appear at least twice in its base-10 representation. For example, 1102021 is redundant, but 11223 is not. Given that 2020 is the nth smallest redundant number, find n.

12. A rectangular garden is built, and a rectangular walkway with fixed width is built around it. If the rectangles formed by the garden and walkway both have integral width, sides, and diagonals, what is the smallest possible area of the walkway?

13. Define a sequence such that $a_1 = 2$, and for $n \geq 2$, a_n is the smallest possible positive integer greater than a_{n-1} such that there do not exist $1 \leq i, j \leq n$, not necessarily distinct, with $a_i + a_j = a_n$. Compute a_{2020}.

14. Let x and y be real numbers such that $\cos x \neq 0$ and $\tan x = 3^{y+1}$. What is the largest possible value of $\sec x - 9^y$?

15. Serena tells Thomas and Ulysses that she is thinking of a two-digit positive integer. She truthfully and separately tells Thomas the tens digit, from which he can conclude that her number is not a perfect square, and Ulysses the units digit, from which he can also conclude that her number is not a perfect square. What is the sum of all possible numbers Serena is thinking of?

16. Alice flips a fair coin until she flips her third "tails", at which point she stops. Given that she flips the coin at least five times and that the fifth flip was "heads," find the expected number of times that Alice flips the coin.

17. There is a unique increasing sequence $p_1, p_2, \ldots p_k$ of primes for $k \geq 2$ such that $p_k = 13$ and

$$\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_k} - \frac{1}{p_1 p_2 \cdots p_k}$$

is an integer. Compute $p_1 + p_2 + \cdots + p_k$.

18. Let $ABCD$ be a convex quadrilateral with $AB = 6$, $BC = 9$, $CD = 15$, and $DA = 10$. Point P is chosen on line AC so that $PB = PD$. Given that $AC = 11$, find PB.

19. The polynomials

$$P(x) = x^3 - 7x^2 + ax + b$$

and

$$Q(x) = x^5 - 5x^4 + cx^3 + dx^2 + ex + f.$$

have the same set of roots, where a, b, c, d, e, f are real numbers. If $P(3) = 6$, find the sum of all possible values for $Q(3)$.

20. Let ABC be a triangle, and let the excenter opposite A be I_A. Points P, Q lie on the extensions of sides AB, AC past B, C respectively so that $BP = BC = CQ$. Suppose P, I_A, Q are collinear. Given that $AB = 20$ and $AC = 21$, find BC.