Team Round

- **DO NOT open this test until your proctor tells you to begin.**
- This portion of the contest consists of 12 problems that are to be completed in 30 minutes, along with a small minigame that will generate a multiplier for a team's team round score.
- To ensure that your answers to problems 1 to 12 are marked correct if they are indeed correct, be sure that your answers are *simplified* and *exact*. Carry out any reasonable calculations (unless the answer obtained is greater than 10^{10}). Write fractional answers in the form $\frac{a}{b}$ where a, b are expressions not containing any fractions. Any decimals must be exact; rounded answers will not receive credit. Any square factors inside square roots must be moved outside the radical.
- There is no partial credit or penalty for incorrect answers.
- The minigame, or problem 13, will ask your team to submit a pair of real numbers (x, y), and will generate a multiplier between 0.9 and 1.1 for your team round score. Be sure to try out the minigame, as submitting nothing will decrease your team round score.
- Each of the 12 team round problems have a predetermined point value; your team's team round score will be the sum of the point values assigned to each question that is correctly answered, multiplied by the multiplier generated by the minigame. Excluding the multiplier, a perfect score (achieved through answering all 12 problems correctly) is **400 points**.
- Your team score will be a combination of your score on the team round and the scores of each individual member.
- No aids other than the following are permitted: scratch paper, graph paper, ruler, compass, protractor, writing utensils, and erasers. **No calculators or other electronic devices (including smartwatches) are permitted.**
- Please make sure to record your team name, team members' names, and all answers on your answer form. Only the responses on the answer forms will be graded.

1. **[22]** How many ways can Danielle select two pets of different color from five brown dogs, seven grey kittens, and eight yellow parakeets? Two animals of the same color are still considered distinguishable.

2. **[24]** Find the unique three-digit positive integer which
   - has a tens digit of 9, and
   - has three distinct prime factors, one of which is also a three-digit positive integer.

3. **[26]** A bin of one hundred marbles contains two golden marbles. Shen, Li, and Park take turns, in that order, removing marbles from the bin. If a player draws a golden marble, the game ends and the player wins. What is the probability that Shen wins?

4. **[28]** Points $A(5,1)$, $B(1,7)$, $C(3,10)$, and $D(7,10)$ are on the coordinate plane. Circle γ is tangent to $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$. What is the area of γ?

5. **[30]** Jerry picks positive integers a,b,c. It turns out that $\gcd(a,b)=\gcd(b,c)=\gcd(c,a)=1$ and the cubic x^3-ax^2+bx-c has distinct integer roots. What is the smallest possible value of $a+b+c$?

6. **[32]** Compute $\sqrt{73^3-2^4\cdot 3^7}$, given that it is an integer.

7. **[34]** Nine fair coins are flipped, and the coins are randomly divided into three piles of three coins. What is the expected number of heads in the pile with the largest number of heads?

8. **[36]** What is the sixth smallest positive real x such that $x\cdot\lfloor x\rfloor\cdot\{x\}=6$? (Here, $\lfloor x\rfloor$ and $\{x\}$ denote the integer and fractional parts of x, respectively.)

9. **[38]** Triangle ABC has $AB=6$, $BC=11$, and $CA=7$. Let M be the midpoint of $\overline{BC}$. Points E and O are on $\overline{AC}$ and $\overline{AB}$, respectively, and point N lies on line AM. Given that quadrilateral $NEMO$ is a rectangle, find its area.

10. **[40]** Daniel paints each of the nine triangles in the diagram either crimson, scarlet, or maroon. Given that any pair of triangles sharing a side are painted different colors, how many ways can Daniel paint the diagram? Two diagrams that differ by a rotation or reflection are considered distinct.

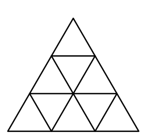
11. **[42]** An S-tetromino is inscribed in square $ABCD$ such that their perimeters share exactly four points, one of which is X. Given that A, B, and X are collinear, maximize $\frac{AX}{BX}$. (An *S-tetromino* is a geometric figure comprised of four unit squares, joined edge to edge in the pictured fashion.)

12. **[44]** Let n be a positive integer. Points A,B,C are selected from the unit n-dimensional hypercube

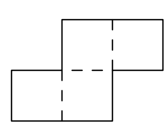
$$\mathcal{H}=\{(x_1,x_2,\dots,x_n)\mid 0\le x_i\le 1 \text{ for } i=1,\dots,n\}.$$

   Given that the maximum possible area of $\triangle ABC$ is a positive integer, minimize n.

13. Pick a point $P=(x,y)$ such that $0\le x,y\le 1$. Let S be the set of points submitted by all teams, $f(A)$ be the minimum distance from A to any other point in S, and let D be the maximum value of $f(A)$ across all points A in S. Your multiplier will be $0.9+0.01\times\left\lfloor\frac{20f(P)}{D}\right\rfloor$. If you submit nothing or pick an invalid point, your multiplier will default to 0.9. In particular, you would like your point to be as far away from the closest other submitted point as possible.



Problem 10.



Problem 11.