Team Round

- **DO NOT open this test until your proctor tells you to begin.**
- This portion of the contest consists of 12 problems that are to be completed in 30 minutes, along with a small minigame that will generate a multiplier for a team's team round score.
- To ensure that your answers to problems 1 to 12 are marked correct if they are indeed correct, be sure that your answers are *simplified* and *exact*. Carry out any reasonable calculations (unless the answer obtained is greater than 10^{10}). Write fractional answers in the form $\frac{a}{b}$ where a, b are expressions not containing any fractions. Any decimals must be exact; rounded answers will not receive credit. Any square factors inside square roots must be moved outside the radical.
- There is no partial credit or penalty for incorrect answers.
- The minigame, or problem 13, will ask your team to fill in an 8×8 grid satisfying certain conditions, and will generate a multiplier for your team round score. Be sure to try out the minigame, as failure to meet the conditions (even if nothing is submitted) may decrease your team's team round score.
- Each of the 12 team round problems have a predetermined point value; your team's team round score will be the sum of the point values assigned to each question that is correctly answered, multiplied by the multiplier generated by the minigame. Excluding the multiplier, a perfect score (achieved through answering all 12 problems correctly) is **400 points**.
- Your team score will be a combination of your score on the team round and the scores of each individual member.
- No aids other than the following are permitted: scratch paper, graph paper, ruler, compass, protractor, writing utensils, and erasers. **No calculators or other electronic devices (including smartwatches) are permitted.**
- Please make sure to record your team name, team members' names, and all answers on your answer form. Only the responses on the answer forms will be graded.

1. **[22]** Determine the smallest positive integer n such that $(n+1)^{n+2} - n^{n+1}$ is composite.

2. **[23]** Compute
$$\left\lfloor \frac{\binom{2018}{20}}{\binom{2018}{18}} \right\rfloor.$$

3. **[25]** For the mock PUMaC competition, Ben wants to assign two subjects out of algebra, combinatorics, geometry, and number theory to each of four team members such that each subject is assigned to exactly two members. How many ways are there to do this?

4. **[28]** In rectangle $ABCD$ with $AB = 8$ and $BC = 10$, cut four 1×1 squares off the corners, thus forming a 12-sided polygon with eight 90° angles and four 270° angles. To this polygon, add 4 quarter circles with radii 1, centers at the 270° angles, and oriented so that they don't overlap with the polygon. Compute the radius of the smallest circle that can circumscribe this new convex figure.

5. **[30]** Five real numbers are selected independently and at random from the interval $(0, 1)$. Let m be the minimum of the five selected numbers. What is the probability that $\frac{1}{4} < m < \frac{1}{2}$?

6. **[32]** In right triangle $\triangle ABC$ with $\angle BAC = 90^\circ$, the incircle ω touches BC, CA, and AB at D, E, and F, respectively. Let $G \neq E$ be the second intersection of BE with ω. If DG and AC are parallel and the radius of ω is 1, compute the area of $\triangle ABC$.

7. **[35]** Determine the minimum possible value of $(x+y)^2 - 2(x+2)(y+2)$ for real numbers x and y.

8. **[37]** How many ordered triples (x, y, z) of real numbers are there such that
$$xy + z = xz + y = yz + x = 0?$$

9. **[38]** For a positive integer n, let $d(n)$ be the number of divisors of n. Find the sum of all n such that
$$n = 7d(n).$$

10. **[42]** I am playing a 5 question trivia game in which I don't know any of the answers. However, the game gives me a list of the 5 correct answers, all distinct, to the questions in a random order. Every time I guess an answer to a question, I will find out whether or not I got it right before moving on to the next question. I guess the first answer on the list until I get a question correct, then I guess the second answer until I get another question correct, and so on, until I run out of questions. What is the expected value of the number of questions I will answer correctly?

11. **[43]** In square $ABCD$ with side length 6, let M and N denote the midpoints of BC and AD. A circle ω with radius 1 has center O contained on line MN. A tangent from A to ω and a tangent from B to ω intersect at a point P on line CD, so that ω lies inside ABP. What is $AP + BP$?

12. **[45]** Compute the sum of all fractions $0 < \frac{a}{b} < 1$ with a decimal representation that is periodic immediately after the decimal point with smallest period 6.

13. Let the number of people taking NEMO this year be an integer X. Estimate X. The multiplier to your team score will be $0.9 + 0.01 \left\lfloor 20 \min\left(\frac{X}{Y}, \frac{Y}{X}\right)\right\rfloor$ where Y is the answer you submit. (If your team submits nothing or a non-positive answer, your multiplier will default to 0.9.)